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Mortgage Loans and Bank Risk Taking: Finding the Risk "Sweet Spot"

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A vast body of academic literature deals with banks' optimal loan allocations. The general approach to solving this problem is to assume borrowers' portfolios as given. Although this assumption is reasonable in the corporate sector, the situation differs radically in the mortgage markets, where borrowers are unobservable and banks' screening capacity is tightly limited. We propose a novel *dynamic model* that assumes potential mortgage takers arrive randomly and sequentially at a bank. In a simulation, we show that the effect of a more stringent level of perceived risk on a bank's expected net income can be positive or negative. Remarkably, if both level of wealth inequality and screening capacity are low, a more severe level of perceived risk can decrease a bank's expected net income. In this situation, regulators should be particularly careful about increasing regulation in the form of a lower loan-to-value ratio.

Keywords: Mortgage loans; stress testing; wealth distribution; screening capacity; power law distribution.

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1. Introduction

There is an extremely large body of literature, both empirical and theoretical, that deals with banks' risk taking. Given many publications could fit this description, it is impossible to address them all here.[1] One important attribute shared across this literature is the common treatment of a bank risk-taking problem as a kind of portfolio allocation problem (see, among others, Gollinger and Morgan, 1993; Allen and Gale, 2000; Hellman *et al.*, 2000; Bessis, 2015). This assumption is reasonable in the context of corporate loans or private banking. In these sectors, banks actually know, or can know, their clients fairly well; moreover, in many cases, there is a repeated game situation — corporations tend to turn to the same credit providers time after time. The situation is completely different in the mortgage sector. The customers arrive at banks to get quotes almost randomly, and households do not take mortgages on a daily basis.[2] In turn, mortgage credit officers, mostly, do not have adequate data about their potential clients. Furthermore, screening capacity[3] is relatively tight in this sector. In these circumstances, banks face two types of risks. On the one hand, there is a credit risk inherent in risky mortgages. This risk provides an incentive to refrain from offering large mortgages. On the other hand, because borrowers arrive randomly, a bank risks not being successful in loaning its allotted funds. The latter risk drives the bank to provide larger mortgages. Thus, the banks have to try to find the "sweet spot" between these two types of risk.

In this paper, we have proposed a novel approach in which we have modeled this real-world risk environment. More specifically, we explore the effects of changes in the level of a bank's perceived risk on the expected net income of that bank, the total amount of loans provided by the bank, and the probability that the bank owners' equity would be eradicated. By "perceived risk", we mean a bank manager's pessimistic assessment of the future state of the economy, which can be quantified as a reduction in the future value of the

[1] For a review, see, for example, Gorton and Winton (2003).

[2] Housing is the most important asset in the portfolio of most households. In addition to the important decision to purchase a house, decisions about how to finance it must be made. Understanding the decision-making process of home owners has important implications from a policy perspective, because of the effects it may have on housing prices, housing market stability, and household welfare. For a psychological explanation of the household choice between variable- and fixed-rate mortgages, see Mugerman *et al.* (2016).

[3] The capacity to screen borrowers, also called capacity constraint. Screening capacity is defined here as a bank's capability to service a given number of borrowers at a certain point in time.

borrower's collateral assets.[4] Important components of our model, in addition to the perceived risk,[5] are borrowers' wealth inequality (i.e., the inequality of wealth distribution across borrowers) and the bank's screening capacity. Therefore, if the bank's perceived risk is more severe, the bank will lend a lower amount for the same current value of assets. In other words, when the bank's perceived risk is higher, the internal loan-to-value ratio will be lower.

We assume that the regulator determines the maximum loan-to-value ratio, either directly by setting a specific ratio or indirectly through a system of stress testing requirements. In a sense, the regulator sets some minimal level of pessimism and the decision maker in the bank cannot be less pessimistic than that level. In Sec. 2, we quantify the relationship between the regulator's loan-to-value ratio (or/and the stress testing requirements) and the level of perceived risk as adopted by the bank.

To the best of our knowledge, no previous study has addressed the interdependencies between the level of perceived risk, the inequality of borrowers' wealth, and the properties of the distribution of a bank's net income. This article is related to several strands in the literature: stress testing, screening quality or capacity, and inequality of wealth distribution across borrowers. The topics of perceived risk and the distribution of wealth have been discussed in the literature independently. Similarly, the literature does not consider the interaction between perceived risk and the screening capacity of banks.

Given concerns related to the economic costs and political ramifications of a bank failure, a country's central bank regulator is responsible for the prevention of any situation in which the owners' equity in a bank is eliminated. Accordingly, the regulator establishes a minimum level of equity such that the probability of the loss of the total value of the equity is sufficiently small. In reality, the regulator does not know the exact statistical distribution of the profit (loss) of a bank; hence, the central bank regulator relies on some type of worst-case analysis to identify the adequate level of capital. Loan-to-value regulatory restrictions, or their more broadly defined counterparts, stress test analyses, seem to have stemmed from the latter point.

[4] The notion of value refers to the net realizable value of the collateral assets (i.e., all of the sources of economic value from which the bank can recover its funds in case of a loan default, including assets to be purchased by the borrower that are partially financed by the loan). If a borrower wishes to borrow an amount for which its future value exceeds the net realizable value (as estimated by the bank), the loan will be charged a higher interest rate that incorporates the borrower's specific risk premium.

[5] Regulators influence this risk through loan-to-value ratio restrictions and stress testing requirements.

In the wake of the massive financial crisis of 2008, the Dodd–Frank Wall Street Reform and Consumer Protection Act was signed into U.S. federal law in 2010.[6] One of the results of this legislation was the creation of a new regulatory directive: Dodd–Frank Act supervisory stress testing. This directive is a forward-looking quantitative evaluation of the impact of stressful economic and financial market conditions on the capital of bank holding companies (BHCs). The program seeks to inform BHCs, as well as the general public, about the possible changes to institutions' capital ratios during a hypothetical set of adverse economic conditions as designed by the Federal Reserve. In addition to the annual supervisory stress test conducted by the Federal Reserve, each BHC is required to conduct annual company-run stress tests under the same supervisory scenarios and to run a mid-year stress test under company-developed scenarios.

Despite progress in research on banking since 1988, the year of the first Basel Capital Accord, there is still no consensus on the optimal design of bank capital regulation (Santos, 2001). The subprime housing crisis of 2008 prompted regulators to consider the systemic risk associated with the interrelationships between economic conditions, the probability of a borrower's default, and the potential for and scale of bank losses.

The finance literature on stress testing is primarily about the techniques and application problems of stress testing. For example, the Bank for International Settlements (BIS, 2009) detailed the principles for sound stress testing practices and supervision.[7] The Basel Committee suggested that establishing the appropriate buffer size of the owners' equity should be approached via stress testing. Peura and Jokivuolle (2004) described a simulation-based approach to stress testing of regulatory capital adequacy where rating transitions are conditioned on the business-cycle phase, and business-cycle dynamics are taken into account. Their work did not take into account the effect of the required level of stress testing set by the regulator on the bank's loan decision making or the interactions between (i) the level of stress testing, (ii) the properties of borrowers' wealth distribution, and (iii) the screening capacity of the bank and its effects on the owners' equity in the bank.

[6] http://www.gpo.gov/fdsys/pkg/PLAW-111publ203/html/PLAW-111publ203.htm

[7] Santos (2001) provided a comprehensive review of the literature on bank capital regulation. Drehmann (2008) provided a general introduction to stress testing. Other articles include Haldane *et al.* (2007) and Hoggarth and Whitley (2003). Foglia (2009) introduced a survey of approaches by authorities to stress testing credit risk and Quagliariello (2009) presented a collection of papers that discuss stress testing.

Another strand of the stress testing literature focuses on the relationship between the information reported to the public on stress testing results and the effect of this information on securities markets. For example, Morgan *et al.* (2014) investigated whether the Federal Reserve's 2009 examination of the 19 largest U.S. BHCs produced useful information for the market. Their findings are consistent with the view that stress tests produce valuable information about banks for securities market participants. Similarly, Quijano (2014) examined whether the 2009 bank stress test conducted by the Federal Reserve conveyed new information to investors. In a similar vein, Petrella and Resti (2013) examined the 2011 European stress test exercise to assess whether and how it affected bank stock prices. Their analysis shows that stress tests produce valuable information for market participants and can help mitigate bank opacity. The current study does not refer to these market effects.

Other studies have focused on stress testing an integrated risk, which is composed of credit risk and a specified other risk. For example, Drehmann *et al.* (2010) derived a consistent and comprehensive framework to measure the integrated impact of both credit and interest rate risk (IRR). Similarly, Abdymomunov and Gerlach (2014) proposed a new method for generating yield-curve scenarios for stress testing banks' exposure to IRR. They argued that their method provides more information about the bank's exposure to IRR using fewer yield-curve scenarios than alternative historical and hypothetical methods. Hartmann (2010) summarized new research on the interaction of market and credit risk and implications for risk management. Our research focuses specifically on credit risk, where the borrower's default probability depends on subjective borrower characteristics and the state of the economy. Breuer *et al.* (2012) proposed a new method for the more systematic analysis of multi-period stress scenarios for portfolio credit risk than current macro stress tests. A network theory approach to stress testing was presented by Levy-Carciente *et al.* (2015). They developed a dynamic model using a bipartite network model of banks and their assets to analyze the system's sensitivity to external shocks. They applied this model in a stress test of the Venezuelan banking system.

In response to the question of whether more stringent perceived risk is good for banks, we found mixed results: More stringent perceived risk decreases the probability of a total loss of equity. However, it also decreases the total amount of loans provided by the bank to borrowers. When wealth is more equally distributed and screening capacity is low, it becomes harder for the bank to lend its funds. Under these circumstances, if the central bank regulator imposes a more stringent level of stress testing, it may lead to a decrease in the average net income of the bank. Hence, the regulator in each country should tailor the proper level of stress testing to each bank given its specific screening capacity and the distribution of wealth across its potential borrowers.

The finance literature does not include studies of the notion of screening capacity. Rather, the focus is on the effects of proprietary information on the banking system (Marquez, 2002; Banerjee, 2005; Hauswald and Marquez, 2006; Loutskina and Strahan, 2011; Purnanandam, 2011), on the relationship between business cycles and the profitability of screening (Ruckes, 2004), and on screening quality (Murfin, 2012).

On the topic of wealth inequality, Peress (2004) showed that because information generates increasing returns, decreasing absolute risk aversion, in conjunction with the availability of costly information, is sufficient to explain why wealthier households invest a larger fraction of their wealth in risky assets. He reported that the availability of costly information exacerbates the negative effects of wealth inequalities. Iacoviello (2008) constructed an economy with heterogeneous agents that mimics the time series behavior of the earnings distribution of U.S. households from 1963 to 2003. He illustrated how this model economy replicates two key features of the distribution: the trend and cyclical behavior of household debt and the diverging patterns in consumption and wealth inequality over time. In this context, in our study, we examine the effects of perceived risk while also considering its interactions with the level of screening capacity and the inequality in the distribution of borrowers' wealth.

This paper is organized as follows. In Sec. 2, we introduce our formal model. Section 3 presents the methodology of the research. Section 4 presents the simulation results and analyses and Sec. 5 provides a summary and conclusions.

2. The Model

The model takes into account the level of perceived risk and the screening capacity of the bank. A systemic risk is integrated into the model by assuming that the probability of a borrower's default as well as the fair market value of the collateral assets depends on the random state of the economy. An important characteristic of the model is that borrowers randomly apply to the bank for a loan, while the bank decides whether to grant the loan and whether to loan the entire amount applied for or a lesser amount. We assume a competitive market for loans in the sense that the bank has to make a decision on each loan application in order of arrival and cannot postpone the decision to a later point in time, that is, after having received loan applications from all borrowers. Therefore, for example, if a high-risk borrower applies for a loan, the bank may be reluctant to lend a large amount because of the default risk but must take into account the risk that it may fail to lend all its funds allocated for loans. Thus, the bank may approve a loan in spite of the perceived higher default risk. We assume that when the bank manager makes a decision on the amount of a loan, she takes into account that the state of the economy in the future might get worse and sets the amount of the loan accordingly.

The model includes the following actors: a bank, an owner/manager decision maker, a set of potential borrowers, and the regulator who imposes the loan-to-value ratio (and/or stress testing) on the bank. We assume that the bank earns its net income solely from lending activity. We denote the fair market value of borrower n's collateral assets by $C_n(\delta, \mathbf{a}_n)$, where δ is the continuous and random state of the economy and $\mathbf{a}_n = (a_{n,1}, \ldots, a_{n,m})$ is a vector of m attributes of the specific collateral assets. The fair market value of the collateral assets increases as the state of the economy improves, that is, $\frac{\delta C_n}{\partial \delta} > 0$. For example,[8] we can express $C_n(\delta, \mathbf{a}_n)$ as follows:

$$C_n(\delta, \mathbf{a}_n) = \frac{1}{\pi}\left(\frac{\pi}{2} + \arctan g\delta\right)(C_n^U(\mathbf{a}_n) - C_n^L(\mathbf{a}_n)) + C_n^L(\mathbf{a}_n), \qquad (1)$$

where the upper bound $C_n^U(\mathbf{a}_n)$ and the lower bound $C_n^L(\mathbf{a}_n)$ of the fair market value are determined by the specific attributes of the collateral assets. Note that $\lim_{\delta\to\infty} C_n(\delta, \mathbf{a}_n) = C_n^U(\mathbf{a}_n)$ and $\lim_{\delta\to-\infty} C_n(\delta, \mathbf{a}_n) = C_n^L(\mathbf{a}_n)$. Figure 1 presents the relationship between the state of the economy and the fair market value of the collateral assets of borrower n.

Similarly, we assume that the probability of default of borrower n, denoted by $\lambda_n(\delta, \mathbf{s}_n)$, depends on the state of the economy, and a vector $\mathbf{s}_n = (s_{n,1}, \ldots, s_{n,r})$ is of r attributes of borrower n. The probability of default decreases when the state of the economy improves, that is, $\frac{\partial \lambda_n}{\partial \delta} < 0$. The model is for a single period that includes several intervals. In the first interval of the period, borrowers randomly arrive at the bank and apply for a loan and the bank decides on the amount of the loan and on the risk premium. In the

[8] Any logistic function that obeys a similar pattern will suffice.

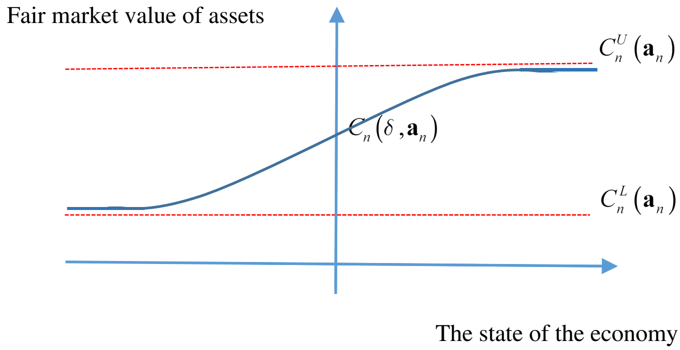

Fig. 1. Relationship between the state of the economy and the fair market value $C_n(\delta, \mathbf{a}_n)$ of the collateral assets of borrower n. $C_n^U(\mathbf{a}_n)$ and $C_n^L(\mathbf{a}_n)$ are the upper and lower bounds of the fair market value.

second interval, the state of the economy δ is realized and in turn, the probability of default $\lambda_n(\delta)$ for each borrower n is realized. At the end of the second interval, it becomes known whether borrower n is in default and the loss or gain to the bank is realized.

2.1. *Beginning of the period*

At the beginning of the period, the state of the economy, δ_0, is (normalized) zero, that is, $\delta_0 = 0$. We assume that the fair market value $C(\delta_0)$ of each borrower's assets or wealth is power law distributed (see Levy and Solomon (1997); Drăgulescu and Yakovenko (2001)). The probability density function of the borrower's wealth is

$$g(x) = \frac{\beta - 1}{a^{1-\beta} - b^{1-\beta}} x^{-\beta}, \quad \text{for } \beta > 1 \quad \text{and} \quad a < x < b, \tag{2}$$

and the cumulative distribution function is given by

$$G(x) = \frac{a^{1-\beta} - x^{1-\beta}}{a^{1-\beta} - b^{1-\beta}}. \tag{3}$$

As β increases, borrowers' average wealth decreases, and the wealth of borrowers is more equally distributed.[9]

Parameter b has two interpretations. It represents the upper bound of the borrowers' wealth distribution and it may represent the conditional Pareto distribution when the regulator imposes a cap on the maximum amount

[9] This effect is derived from power law function and is explained in Appendix A.

that the bank is allowed to lend to any specific borrower. The Pareto distribution is

$$G(x) = 1 - \left(\frac{x}{a}\right)^{1-\beta}, \quad \text{for } \beta > 1 \quad \text{and} \quad x > a. \tag{4}$$

Setting a cap b on the Pareto distribution yields the power law distribution. This follows from the conditional probability

$$\mathrm{Prob}(l \leq x | x < b) = \frac{G(x)}{G(b)}. \tag{5}$$

Because

$$\frac{G(x)}{G(b)} = \frac{1 - \left(\frac{x}{a}\right)^{1-\beta}}{1 - \left(\frac{a}{b}\right)^{1-\beta}},$$

we obtain the power law distribution for $a < x < b$ as in Eq. (3), $\mathrm{Prob}(l \leq x | x < b) = \frac{a^{1-\beta} - x^{1-\beta}}{a^{1-\beta} - b^{1-\beta}}$. In addition, the bank manager determines, in light of the central bank regulator's set level of stress testing, the perceived negative state of the economy at the end of the period, denoted by δ_p.

2.2. *First interval of the period*

During the first interval of the period, borrowers randomly arrive at the bank and apply for a loan. For each borrower n, the bank uses its screening system to estimate the fair market value, $C_n(\delta_0)$, and the net realizable value, $C_n^R(\delta_0)$, of the borrower's collateral assets and estimates the borrower's probability of default, $\lambda_n(\delta_0)$. These estimates are calculated for the state of the economy at the beginning of the period.

The net realizable value is estimated as a fraction φ of the estimated fair market value. That is, $C_n^R(\delta_0) = \varphi \cdot C_n(\delta_0)$, where $\varphi \in [0, 1]$ is random and its distribution depends on the state of the economy. We assume that as the state of the economy improves, the net realizable value of the collateral assets approaches their fair market value. Specifically, φ is uniformly distributed over the range $[\varphi_L(\delta), \varphi_U(\delta)]$, where $\frac{d\varphi_L}{d\delta} > 0$ and $\frac{d\varphi_U}{d\delta} > 0$.

To determine the amount to lend to borrower n, the manager also estimates the above variables for the perceived state of the economy at the end of the period: $C_n(\delta_P)$, $C_n^R(\delta_P)$, and $\lambda_n(\delta_P)$. We assume that the request for a loan is dependent on the borrower's collateral assets. Therefore, it is assumed that the request for a loan by borrower n, denoted by L_n^D, is randomly distributed over the range $L_n^D \in [(1-\psi)\, C_n(\delta_0), (1+\psi)\, C_n(\delta_0)]$ for some $\psi \in [0, 1]$.

We denote the interest rate by i and the weighted average cost of capital by k. Given the request, L_n^D, the manager makes decisions on the size of the loan and on the interest risk premium ρ_n. The minimum loan to borrower n will be the present net realizable value of the collateral assets, $C_n^R(\delta_P)/(1+i)$. When the borrower's loan request is greater than that value, the manager may increase the loan to borrower n if the net expected profit from any additional loan amount is positive. For an additional loan amount of 1 dollar, the net cash flow at the beginning of the period is zero since the bank finances the additional dollar by equity and debt. The net expected cash flow at the end of the period is $(1+\lambda_n(\delta_P))(1+i)+\lambda_n(\delta_p)\cdot 0-(1+k)$. Therefore, the manager will be willing to give an additional loan amount of 1 dollar if $(1-\lambda_n(\delta_P))(1+i)>1+k$. Hence, the manager will increase the amount of the loan if and only if $1-\lambda_n(\delta_P)>\frac{1+k}{1+i}$, or

$$\lambda_n(\delta_P)<\frac{i-k}{1+i}. \tag{6}$$

We denote $\lambda^*=\frac{i-k}{1+i}$ as the threshold probability. If the probability of default given the perceived state of the economy at period end is lower than the designated threshold probability λ^*, the manager will increase the loan.

The manager assumes that the borrower will choose not to repay the loan (even in the case of no default) if the total amount of the loan including interest exceeds the fair value of the collateral assets, $C_n(\delta_p)$. Therefore, the loan amount must not exceed the present fair market value of the collateral assets, assuming the perceived adverse state of the economy at the end of the period. In other words,

$$L_n\leq\frac{C_n(\delta_P)}{1+i}. \tag{7}$$

Denote the level of the bank's perceived risk by p, such that $-1<p<0$. We assume that

$$C_n(\delta_P)=(1+p)\,C_n(\delta_0). \tag{8}$$

By Eqs. (7) and (8), it must be that

$$\frac{L_n}{C_n(\delta_0)}\leq\frac{1+p}{1+i}. \tag{9}$$

The relationship of the loan-to-value ratio as set by the regulator and the internal perceived risk adopted by the bank is as follows. Denote by η the maximum level of the loan-to-value ratio as set (directly or indirectly) by the regulator. That is, per the regulation, for any borrower n,

$$\frac{L_n}{C_n(\delta_0)} \leq \eta. \tag{10}$$

Since the internal ratio cannot exceed the ratio set by the regulator, following Eqs. (9) and (10) it must be that

$$\frac{1+p}{1+i} \leq \eta. \tag{11}$$

Thus, the perceived risk must obey[10]

$$p \leq (1+i)\eta - 1. \tag{12}$$

To sum up, according to Eq. (7) the amount of the loan, L_n, to borrower n is expressed by

$$L_n = \begin{cases} \mathrm{Min}\left(L_n^D, \dfrac{C_n(\delta_P)}{1+i}\right) & \text{if } \lambda_n(\delta_P) < \lambda^* \\ \mathrm{Min}\left(L_n^D, \dfrac{C_n^R(\delta_P)}{1+i}\right) & \text{if } \lambda_n(\delta_P) \geq \lambda^* \end{cases}. \tag{13}$$

If $\lambda_n(\delta_P) < \lambda^*$ and the loan is higher than the net realizable value of the collateral assets, that is, $L_n > C_n^R(\delta_P)$, the manager adds a risk premium ρ_n to the interest rate i. The purpose of adding the risk premium is to guarantee that the net effective rate of interest remains i given the expected default. The risk premium is given by[11]

$$\rho_n = \frac{\lambda_n(\delta_P)}{1-\lambda_n(\delta_P)}\left(1 - \frac{C_n^R(\delta_P)}{L_n} + i\right), \tag{14}$$

and the total interest charged on borrower n is $i_n = i + \rho_n$.

There are certain constraints during the first interval of the period. When a borrower arrives at the bank and wants to apply for a loan, the borrower's application will be processed if the bank has not reached its screening capacity constraint, denoted by SC. We assume that the screening capacity of the bank has a cost of $F_{\mathrm{SC}} = a \cdot \mathrm{SC}^{\gamma}$, where a is a constant coefficient and $0 < \gamma < 1$ is a measure of the elasticity of the screening capacity cost.

The model assumes structural elasticity in the sense that the bank can increase or decrease its owners' equity and its debt. However, there are upper

[10] For example, if the loan-to-value ratio, η, is equal to 0.8 and the interest rate is 10%, then the level of perceived risk must not exceed (−12%).
[11] The background for this equation is presented in Appendix B.

and lower limits for such changes. For example, assume that at the beginning of the period the total funds available for loans is 20 million dollars, which is composed of 2 million dollars of owners' equity and 18 million dollars of debt (deposits, debentures, etc.). If the sum of loans exceeds 20 million dollars, say, 22 million dollars, the bank can mobilize an additional 2 million dollars, while keeping its weighted average cost of capital, that is, an additional 0.2 million dollars in owners' equity and 1.8 million dollars in debt. Nevertheless, in the simulation we assumed that the bank cannot increase its funds above 30 million dollars.

2.3. *Second interval of the period*

During the second interval of the period, the state of the economy δ is realized and in turn, the probability of default $\lambda_n(\delta)$ for each borrower n is realized. Note that because δ is realized it determines the fair market value, $C_n^F(\delta)$, and the net realizable value, $C_n^R(\delta)$, of borrower n's collateral assets.

2.4. *End of the period*

We denote default by a binomial variable

$$d_n = \begin{cases} 1 & \text{if borrower } n \text{ is in default} \\ 0 & \text{otherwise} \end{cases}. \tag{15}$$

At the end of the period, each d_n is realized, and it becomes known whether borrower n is in default. We denote by δ_n^* the value of the state of the economy for which the value of the collateral assets C_n is equal to the loan including the accrued interest, that is, $C_n(\delta_n^*) = (1+i)L_n$. When the state of the economy worsens ($\delta < \delta_j^*$), the borrower is motivated to default on the loan even if she/he can repay it. In other words, the default is certain, $\lambda_n = 1$. We denote by δ_n^{**} the value of the state of the economy for which the net realizable value, C_j^R, is equal to the loan including the accrued interest; that is, $C_n^R(\delta_n^{**}) = (1+i)L_n$. We divide the set of n borrowers into three subgroups:

- Group B_1 includes all borrowers for whom the net realizable value is equal to or larger than the total loan (principal and accrued interest); that is, $C_n^R \geq L_n(1+i)$;
- Group B_2 includes all borrowers for whom the fair market value is lower than the total loan; that is, $C_n < L_n(1+i)$;
- Group B_3 includes all borrowers for whom the net realizable value is lower than the total loan but the fair market value is higher than the total loan; that is, $C_n^R < L_n(1+i) \leq C_n$.

All borrowers in B_1 prefer to repay the loan and in those cases where they fail to do so, the bank does not incur a loss. In these cases, the bank sells the collateral, withholds sufficient proceeds to repay the loan including interest, and remits the remaining proceeds to the borrower. The borrowers in B_2 prefer to default, even if they have the ability to repay; that is, $\lambda_j = 1$. The bank's loss is $L_n(1+i) - C_n^R$. In B_3 all borrowers prefer to repay the loan, because they observe a fair market value in excess of the total amount of the loan. Nevertheless, if the borrower fails to repay, the bank incurs a loss of $L_n(1+i) - C_n^R$. Therefore, the expected loss for a borrower n in B_3 to the bank is

$$\lambda_n(\delta) \cdot (L_n(1+i) - C_n^R). \tag{16}$$

In summary, borrower n belongs to a subset B_j, where

$$B_j \in \begin{cases} B_2 & \text{if } \delta < \delta_n^* \\ B_1 & \text{if } \delta \geq \delta_n^{**} \\ B_3 & \text{if } \delta_n^* \leq \delta < \delta_n^{**} \end{cases}. \tag{17}$$

Figure 2 presents the position of borrower n in the three borrower subgroups (B_1, B_2, B_3) according to the end-of-period realized state of the economy.

The loss for the bank from bad debt is

$$\text{Loss}(\mathbf{L}, \mathbf{i}, \mathbf{d}, \mathbf{C^R}) = \sum_{n=1}^{m} d_n \text{Max}[(1+i_n)L_n - C_n^R, 0], \tag{18}$$

where $\mathbf{i} = (i_1, \ldots, i_m)$, m is the total number of borrowers, $\mathbf{L} = (L_1, \ldots, L_m)$ is the vector of the m loans made by the bank, $\mathbf{C^R} = (C_1^R, \ldots, C_m^R)$ is the vector of the net realizable values of the assets, $d_n = 1$ if borrower n is in default and zero otherwise, and $\mathbf{d} = (d_1, \ldots, d_m)$.

Note that the bank incurs losses only for loans made to borrowers in subgroups B_2 and B_3. Therefore,

$$\text{Loss}(\mathbf{L}, \mathbf{i}, \mathbf{d}, \mathbf{C^R}) = \sum_{n \in B_2} ((1+i_n)L_n - C_n^R) + \sum_{n \in B_3} d_n((1+i_n)L_n - C_n^R). \tag{19}$$

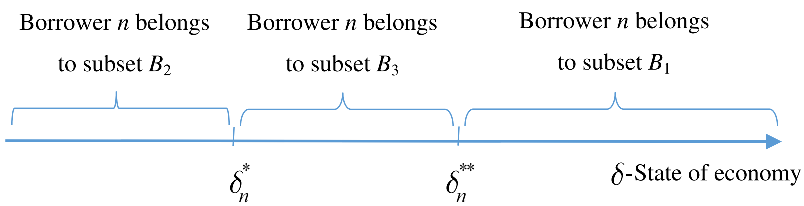

Fig. 2. The state of the economy and the subgroups to which borrower n belongs.

If borrower n belongs to subgroup B_2 ($b_n \in B_2$), she/he prefers to default and the bank loss from the loan to borrower n is $(1 + i_n)L_n - C_n^R$. By contrast, if borrower n belongs to subgroup B_3 ($b_n \in B_3$), she/he prefers to repay the loan but if she/he defaults the bank loss is $(1 + i_n)L_n - C_n^R$. Because some of the borrowers in subgroup B_3 do not default, the total loss from subgroup B_3 is only from defaulted loans. The net income, NI, of the bank is expressed by

$$\mathrm{NI}(k, \mathbf{L}, \mathbf{i}, F, \mathbf{d}, \mathbf{C^R}) = \sum_{n=1}^{m}(L_n(i_n - k)) - F - F_{\mathrm{SC}} - \mathrm{Loss}(\mathbf{L}, \mathbf{i}, \mathbf{d}, \mathbf{C^R}), \quad (20)$$

where F stands for the fixed expenses. Substituting Eq. (19) into Eq. (20) yields

$$\begin{aligned}\mathrm{NI}(k, \mathbf{L}, \mathbf{i}, F, \mathbf{d}, \mathbf{C^R}) = {} & \sum_{n=1}^{m}(L_n(i_n - k)) - F - F_{\mathrm{SC}} - \sum_{n \in B_2}((1 + i_n)L_n - C_n^R) \\ & - \sum_{n \in B_3} d_j((1 + i_n)L_n - C_n^R). \end{aligned} \quad (21)$$

In Eq. (21), there are two random factors: d_n, which is determined by a random variable $\lambda_n(\delta)$, and the division of the set of borrowers into subgroups B_1 and B_2, which is determined by the state of the economy δ.

3. Methodology

The objective of our study is to analyze the effects of the level of perceived risk on the distribution of a bank's net income, while controlling for the level of the borrowers' wealth inequality and the bank's screening capacity. We obtained the following expression for the bank's net income (see Eq. (21)):

$$\begin{aligned}\mathrm{NI}(k, \mathbf{L}, \mathbf{i}, F, \mathbf{d}, \mathbf{C^R}) = {} & \sum_{n=1}^{m}(L_n(i_n - k)) - F - F_{\mathrm{SC}} - \sum_{n \in B_2}((1 + i_n)L_n - C_n^R) \\ & - \sum_{n \in B_3} d_j((1 + i_n)L_n - C_n^R). \end{aligned}$$

Given its complexity, the expression is not tractable to an analytic solution. This complexity derives from the following elements. The binomial variable that determines whether borrower n is in default is itself dependent on the default probability. The default probability is dependent on the random state of the economy. In addition, the distribution of funded loans depends on the distribution of borrowers' wealth, the specific attributes of the borrowers, the screening capacity of the bank, the level of stress testing set by the regulator, and the bank's allotted funds available for loans. In recognition of this complex set of probabilistic interdependencies, we conducted a simulation of the model.

Table 1. Independent controlled variables.

Notation	Description	Min	Max	Delta
Perc	The perceived state of the economy at the end of the period (determined at the beginning of the period)	−0.1	−3.6	−0.25
Max	The upper bound of the distribution of borrowers' wealth	50,000	500,000	50,000
Beta	The exponent of the probability density function of the power law distribution	1.5	2.4	0.1
SC	The bank's capacity to screen borrowers	250,000	3,500,000	250,000

The objective of the simulation was to reach conclusions on the effects of the independent control variables (listed in Table 1) on the dependent variables (Table 2) and any interactions among them using nonmanipulated independent control (exogenous; Table 3) and endogenous (Table 4) variables.

The simulation reflects the randomness of the model's dynamic process. We assume that at the beginning of the period, the final number of borrowers is unknown to the bank. Rather, as each borrower randomly arrives, the bank identifies that borrower's risk of default and the fair market value of the borrower's assets. Then, based on the borrower's loan request and the availability of the bank's funds, the bank decides on the loan amount. This dynamic process terminates at occurrence of the earlier of the following two

Table 2. Dependent variables.

Notation	Description
TL	The sum of total loans
NI	The average net income of the bank
P_r	The probability that the owners' equity in the bank will be eradicated

Table 3. Controlled but not manipulated (exogenous) variables.

Notation	Description	Value
δ_0	State of the economy at the beginning of the period	0
M	Maximum total amount of loans	30,000,000
k	Cost of capital	4%
i	Basic interest rate (before adding risk premium)	9%
F	Fixed cost, $F = F_q + F_{\mathrm{sc}}$, where $F_q = \frac{(i-k)M}{4}$ and $F_{\mathrm{sc}} = 30\mathrm{SC}^{0.6}$	

Table 4. Endogenous random variables.

Notation	Description	Distribution
δ	State of the economy at the end of the period	$\delta \sim N(\mu, \sigma^2)$ is normally distributed, with $\mu = \delta_0 = 0$, and $\sigma = 1$
$C_n(\delta_0)$	The fair market value of the collateral assets of borrower n at the beginning of the period	$C_n(\delta_0) \sim \mathrm{PowerLaw}(1, \mathrm{Max})$
$C_n(\delta)$	The fair market value of the collateral assets of borrower n at the end of the period	$C_n(\delta) = \left(1 + \frac{\delta}{10}\right) C_n(\delta_0)$
$\lambda_n(\delta_0)$	The probability of default at the beginning of the period	$\lambda_n(\delta_0) \in [0\%, 5\%]$
$\lambda_n(\delta)$	The probability of default at the end of the period	$\lambda_n(\delta) = \left(1 - \frac{\delta}{10}\right)\lambda_n(\delta_0)$

events: (i) the bank reaches its maximum screening capacity, or (ii) all funds allocated for lending have been committed. In the case where the process stops because the bank has reached its screening capacity and the total funds for loans have not been fully exploited, the bank adjusts its capital structure so that financial leverage and, in turn, the weighted average cost of capital remain constant.

Let $S = (s_{p=1,\ldots,21{,}000} = (\mathrm{Perc}_i, \mathrm{Max}_j, \mathrm{Beta}_k, \mathrm{SC}_l)), i \in [1, 15], j \in [1, 10], k \in [1, 10], l \in [1, 14]$, be the series of all possible combinations (quadrants) of the independent variables. There are 21,000 of these combinations possible. For every such combination s_p, we executed 5,000 simulations, producing 105,000,000 simulations in total. Performing 5,000 simulations for each combination s_p allowed us to measure the probability that the bank's equity would be eliminated with a precision of $\frac{1}{5{,}000} = 0.02\%$. Each simulation resulted in an average of 1,254,000 borrowers. Therefore, the total sum of all loan simulations was 105,000,000 $\times$ 1,254,000 = 131.67 trillion loans. Each loan was subject to the entire process described in the model section, from initiation to termination.

4. Results of the Simulations

4.1. *Average of the total amount of the loans*

Estimating the effects of the main variables on the average of the total amount of the loans, TL, we obtained the following regression model:

$$\begin{aligned} \mathrm{TL} = {} & 78{,}348{,}355 + 2.297\,\mathrm{Max} - 34{,}241{,}278\,\mathrm{Beta} \\ & + 639{,}821\,\mathrm{Perc} + 3.778\,\mathrm{SC}, \end{aligned} \tag{22}$$

Table 5. Regression of total loans.

	Coefficient	Standard Error	t Statistic	p Value
Intercept	78,348,355	225,717.38	347.11	0
Max	2.297	0.21	11.05	2.65E−28
Beta	(34,241,278)	103,947.60	(329.41)	0
Perc	639,821	27,641.87	23.15	4.6E−117
SC	3.778	0.03	127.52	0

where Max is the upper bound of the distribution of borrowers' wealth, Beta is the exponent of the probability density function of the power law distribution (see Eq. (22)), Perc is the perceived state of the economy at the end of the period (determined at the beginning of the period), and SC is the screening capacity of the bank. All the regression coefficients were statistically significant, $p < 10^{-27}$, and the adjusted R^2 was 85.66% (Table 5).

Max has a positive effect on TL. When Max increases, the expected average of the borrowers' wealth increases, and hence the average TL increases. Beta has a negative effect on TL, such that when Beta increases, the level of equality of wealth increases. Thus, the average level of wealth decreases and, in turn, the total amount of loans decreases. Perc has a negative effect on TL. Since Perc has negative values, when the absolute value of Perc increases (i.e., a more stringent level of stress testing is imposed by the regulator), the average TL decreases. This is because a more stringent level of perceived risk decreases the amount of the loan to each borrower. SC has a positive effect on TL. When SC increases, TL increases. If the bank has a large SC it can process more loan applications simultaneously and, therefore, provide more loans.

From the raw data, we noticed that there were interaction effects among these variables. Therefore, we first ran another regression model that included all the possible interactions. After eliminating all the variables that were not statistically significant, we obtained the following regression:

$$\begin{aligned}\mathrm{TL} = {} & 86{,}616{,}894 + 4.196\,\mathrm{Max} - 38{,}749{,}424\,\mathrm{Beta} - 990{,}337\,\mathrm{Perc} - 2.240\,\mathrm{SC} \\ & - 1.01 \cdot 10^{-6}\,\mathrm{Max} \cdot \mathrm{SC} + 835{,}971\,\mathrm{Beta} \cdot \mathrm{Perc} + 3.229\,\mathrm{Beta} \cdot \mathrm{SC}. \end{aligned} \tag{23}$$

All the regression coefficients were statistically significant, $p < 10^{-6}$, and the adjusted R^2 was 86.4% (Table 6).

The effect of Perc on TL can be seen by its coefficient in Eq. (23): $(-990,337 + 835,971\mathrm{Beta}) > 0$. The overall effect of a more severe level of Perc is a decrease in the average TL. As Beta increases, this negative effect on TL becomes stronger. In other words, when the level of wealth equality as

Table 6. Regression of total loans with interactions.

	Coefficient	Standard Error	t Statistic	p Value
Intercept	86,616,894	555,284	155.987	0
Max	4.196	0.428	9.811	1.13E−22
Beta	(38,749,424)	275,330	(140.738)	0
Perc	(990,337)	184,757	(5.360)	8.4E−08
SC	(2.240)	0.206	(10.897)	1.42E−27
Max*SC	−1.01E−06	2.01E−07	(5.041)	4.67E−07
Beta*Perc	835,971	93,736	8.918	5.11E−19
Beta*SC	3.229	0.100	32.142	2.6E−221

represented by Beta is higher, the regulator should be careful about imposing a more stringent level of stress testing on the bank, as this can further decrease the average amount of loans provided.

4.2. *The average net income*

By estimating the expected net income, NI, we obtained the following regression:

$$\mathrm{NI} = 2{,}083{,}242 + 0.0909\,\mathrm{Max} - 1{,}325{,}938\,\mathrm{Beta} - 249{,}612\,\mathrm{Perc} + 0.0838\,\mathrm{SC}.$$

All the regression coefficients were statistically significant, $p < 10^{-08}$, and the adjusted R^2 was 69.4% (Table 7).

Max has a positive effect on average NI. Despite the small effect of Max, when Max increases, average wealth increases and in turn NI increases. As Beta increases, average wealth decreases and in turn average NI decreases significantly. As Perc decreases, average NI increases. In other words, a more stringent level of stress testing imposed by the regulator has a positive effect on average NI. As SC increases, average NI increases.

On reviewing the raw data, we observed interactions among these variables. Therefore, we ran another regression using a model that included all

Table 7. Regression of the average net income.

	Coefficient	Standard Error	t Statistic	p Value
Intercept	2,083,242	16,427	126.82	0
Max	0.0909	0	6.010059	1.89E−09
Beta	(1,325,938)	7,565	−175.277	0
Perc	(249,612)	2,012	−124.084	0
SC	0.08377	0.002156173	38.85	0

possible interactions. After eliminating all variables that were not statistically significant, we obtained the following regression:

$$\begin{aligned}\mathrm{NI} = {} & 729{,}133 + 0.0922\,\mathrm{Max} - 536{,}511\,\mathrm{Beta} - 1{,}188{,}026\,\mathrm{Perc} - 0.2189\,\mathrm{SC} \\ & - 0.0398\,\mathrm{Max}\cdot\mathrm{Perc} - 3.989\cdot 10^{-8}\,\mathrm{Max}\cdot\mathrm{SC} + 538{,}298\,\mathrm{Beta}\cdot\mathrm{Perc} \\ & + 0.11\,\mathrm{Beta}\cdot\mathrm{SC} - 0.0535\,\mathrm{Perc}\cdot\mathrm{SC}. \end{aligned} \tag{24}$$

All the regression coefficients were statistically significant, $p < 0.007$, and the adjusted R^2 was 79.4%, an increase of 10% over the previous model that excluded the interactions (Table 8).

When examining the relationship between average NI and average amount of TL we found that the variance in TL explained more than 50% of the variance in NI (statistically significant; $p = 0$). This result is intuitive given that providing loans is the core business of the bank. Therefore, we added the error term $\mathrm{TL}_E = \mathrm{TL} - \widehat{\mathrm{TL}}$, as an explanatory variable to NI, where $\widehat{\mathrm{TL}}$ is the estimated average of the total loans in Eq. (23). We obtained the following regression:

$$\begin{aligned}\mathrm{NI} = {} & 737{,}336 + 0.1042\,\mathrm{Max} - 536{,}512\,\mathrm{Beta} - 1{,}183{,}592\,\mathrm{Perc} - 0.2251\,\mathrm{SC} \\ & - 0.0333\,\mathrm{Max}\cdot\mathrm{Perc} - 3.99\cdot 10^{-8}\mathrm{Max}\cdot\mathrm{SC} - 0.0568\,\mathrm{Perc}\cdot\mathrm{SC} \\ & + 538{,}298\,\mathrm{Beta}\cdot\mathrm{Perc} + 0.11\,\mathrm{Beta}\cdot\mathrm{SC} + 0.0385\,\mathrm{TL}_E. \end{aligned} \tag{25}$$

All the regression coefficients were statistically significant, $p < 10^{-05}$, and the adjusted R^2 was 87.5%, an addition of about 8% to the previous model that excluded TL_E (Table 9).

The effect of Perc can be analyzed by the derivative of NI with respect to Perc: $-1{,}183{,}592 - 0.0333\,\mathrm{Max} - 0.0568\,\mathrm{SC} + 538{,}298\,\mathrm{Beta}$. The value and

Table 8. Regression of the average net income with interactions.

	Coefficient	Standard Error	t Statistic	p Value
Intercept	729,133	35,014	20.82	2.41E−95
Max	0.0922	0.033767	2.73	0.006355
Beta	(536,511)	16,884	(31.78)	1.8E−216
Perc	(1,188,026)	12,157	(97.72)	0
SC	(0.2189)	0.012966	(16.89)	1.51E−63
Max*Perc	(0.0398)	0.011496	(3.46)	0.000543
Max*SC	(0.0000)	0.00000001	(3.24)	0.001207
Beta*Perc	538,298	5,748	93.65	0
Beta*SC	0.1101	0.006161	17.87	6.72E−71
Perc*SC	(0.0535)	0.001638	(32.66)	2.7E−228

Table 9. Regression of the average net income with interactions and deviation from expected total loans.

	Coefficient	Standard Error	t Statistic	p Value
Intercept	737,336	27,239	27.07	1.2E−158
Max	0.10415	0.02627	3.96	7.38E−05
Beta	(536,512)	13,135	(40.85)	0
Perc	(1,183,592)	9,458	(125.15)	0
SC	(0.22507)	0.01009	(22.31)	5.1E−109
Max*Perc	(0.03329)	0.00894	(3.72)	0.000198
Max*SC	−3.99E−08	9.59E−09	(4.16)	3.17E−05
Perc*SC	(0.05683)	0.00127	(44.58)	0
Beta*Perc	538,298	4,472	120.38	0
Beta*SC	0.11009	0.00479	22.97	2.4E−115
TL_E	0.03854	0.00033	117.01	0

sign of the derivative are presented in Table 10, for a fixed value of Max = 400,000.

When Beta decreases and SC increases, the effect of Perc on NI is more positive. In other words, when the level of wealth inequality increases, so does the number of wealthy borrowers. Simultaneously, when the bank has a larger SC it is able to be more selective from among these borrowers. Therefore, a more stringent level of stress testing can lead to an increase in average NI. Conversely, when the level of inequality decreases (Beta > 2.2) and the SC of the bank decreases, a more stringent level of stress testing prevents the bank from providing loans and thus decreases the average NI of the bank.

Table 10. The effect of change in the perceived state of the economy at the end of the period (determined at the beginning of the period).

Beta	Screening Capacity							
	250,000	500,000	750,000	1,000,000	1,250,000	1,500,000	1,750,000	2,000,000
1.5	(406,995)	(421,195)	(435,395)	(449,595)	(463,795)	(477,995)	(492,195)	(506,395)
1.6	(353,165)	(367,365)	(381,565)	(395,765)	(409,965)	(424,165)	(438,365)	(452,565)
1.7	(299,335)	(313,535)	(327,735)	(341,935)	(356,135)	(370,335)	(384,535)	(398,735)
1.8	(245,506)	(259,706)	(273,906)	(288,106)	(302,306)	(316,506)	(330,706)	(344,906)
1.9	(191,676)	(205,876)	(220,076)	(234,276)	(248,476)	(262,676)	(276,876)	(291,076)
2	(137,846)	(152,046)	(166,246)	(180,446)	(194,646)	(208,846)	(223,046)	(237,246)
2.1	(84,016)	(98,216)	(112,416)	(126,616)	(140,816)	(155,016)	(169,216)	(183,416)
2.2	(30,186)	(44,386)	(58,586)	(72,786)	(86,986)	(101,186)	(115,386)	(129,586)
2.3	23,643	9,443	(4,757)	(18,957)	(33,157)	(47,357)	(61,557)	(75,757)
2.4	77,473	63,273	49,073	34,873	20,673	6,473	(7,727)	(21,927)

Table 11. The regression of the probability that the owners' equity in the bank would be wiped out, P_r.

	Coefficient	Standard Error	t Statistic	p Value
Intercept	−0.42595	0.01203698	−35.3872	1.8E−266
Max	−1.7E−08	1.10865E−08	−1.51967	0.128608
Beta	0.461867	0.005543282	83.32012	0
Perc	0.035949	0.001474076	24.38742	1.5E−129
SC	−1.3E−07	1.57998E−09	−83.0921	0

4.3. *The probability that the owners' equity will be eliminated*

We defined the event "owners' equity is wiped out" as occurring when NI is less than 10% of TL. By estimating the probability that the owners' equity in the bank would be wiped out, P_r, we obtained the following regression:

$$P_r = -0.42595 - 1.7 * 10^{-8}\,\text{Max} + 0.461867\,\text{Beta} + 0.035949\,\text{Perc} - 1.3 * 10^{-7}\,\text{SC}. \tag{26}$$

The adjusted R^2 is 40.7% (Table 11).

Note that an increase in Max decreased P_r but this effect was not statistically significant. This is because for Beta greater than 2, the effect of increasing Max on the expected wealth is almost nil. An increase in Beta increases P_r. In other words, when the level of wealth equality increases, the probability that the bank's equity is eliminated increases. When the absolute value of Perc increases, P_r decreases. That is, a more stringent level of perceived risk will generally decrease the number of bank closures. Finally, an increase in SC decreases P_r. This is because a higher screening capacity, although costly, enables the bank to make more loans.

From the raw data, we noticed that there were interaction effects among the aforementioned variables. Therefore, we tested another regression model that included all of the possible interactions. After eliminating all variables that were not statistically significant, we obtained the following regression:

$$\begin{aligned} P_r = & -2.11334 + 1.306689\,\text{Beta} + 0.053522\,\text{Perc} + 8.05 * 10^{-7}\,\text{SC} \\ & - 2.7 * 10^{-8}\,\text{Max}\cdot\text{Beta} + 1.87 * 10^{-14}\,\text{Max}\cdot\text{SC} + 1.43 \cdot 10^{-8}\,\text{Perc}\cdot\text{SC} \\ & - 0.02278\,\text{Beta}\cdot\text{Perc} - 4.7 * 10^{-7}\,\text{Beta}\cdot\text{SC}. \end{aligned} \tag{27}$$

All the regression coefficients were statistically significant, $p < 0.002$, and the adjusted R^2 was 61.6%, an addition of more than 20% over the previous model that excluded the interactions (Table 12).

Table 12. The regression of the probability that the owners' equity in the bank would be wiped out, with interactions.

	Coefficient	Standard Error	t Statistic	p Value
Intercept	−2.11334	0.024263392	−87.1001	0
Beta	1.306689	0.012396947	105.4041	0
Perc	0.053522	0.008436566	6.344034	2.28E−10
SC	8.05E−07	9.29694E−09	86.5727	0
Max*Beta	−2.7E−08	9.23026E−09	−2.88725	0.00389
Max*SC	1.87E−14	8.54675E−15	2.193659	0.028271
Perc*SC	1.43E−08	1.17742E−09	12.15893	6.69E−34
Beta*Perc	−0.02278	0.004131111	−5.5133	3.56E−08
Beta*SC	−4.7E−07	4.42772E−09	−105.955	0

The effect of Perc on P_r can be analyzed by using the derivative of P_r with respect to Perc: $0.053522 + 1.43 \cdot 10^{-8}\,\mathrm{SC} - 0.02278\,\mathrm{Beta} > 0$. Within the range of our simulation, the effect of a more stringent level of perceived risk was positive in the sense that it decreased the probability that the bank owners' equity would be eliminated. However, increasing the screening capacity of the bank increased the positive effect of a more stringent level of perceived risk, and increasing the level of wealth equality decreased the positive effect of a more stringent level of perceived risk. Therefore, when a government adopts a policy that increases wealth equality, if bank regulators want to uphold the positive effect of a more stringent level of perceived risk they should take steps to increase the screening capacity of banks.

5. Summary and Conclusions

The central bank determines the maximum loan-to-value ratio, either directly by setting a specific number, or indirectly, as implied by stress testing requirements. In turn, the bank's manager decides on the level of perceived risk, which is constrained by the regulatory loan-to-value ratio.

Our results show that the effect of an increase in the perceived risk on the bank's expected net income could be either positive or negative. It is also intuitively appealing that the results support that a more stringent level of perceived risk decreases both the expected amount of total loans and the probability that the owners' equity in the bank will be eliminated. Table 13 presents the sign of the partial derivative of each dependent variable with respect to the perceived state of the economy at the end of the period as obtained in the simulation.

Table 13. Signs of the partial derivatives.

Dependent Variable	Perc
TL	+
NI	+/−
P_r	+

With respect to the effects of Perc on the average NI of the bank, our results show that when wealth is more equally distributed, and SC is low, it becomes harder for the bank to lend its funds. Under these circumstances, a more stringent level of perceived risk may lead to a decrease in the average NI of the bank. Table 14 summarizes the effects of Perc on the average NI of the bank.

With respect to the effects of Perc on the average TL, our results show that when the level of perceived risk increases, the average amount of total loans decreases. This effect becomes stronger when the level of wealth equality is higher. Under these circumstances, the regulator should recognize that imposing a more stringent level of perceived risk on the bank may lead to a further decrease in the total amount of loans provided. Table 15 summarizes the effects of Perc on the average TL.

Table 14. The effects of the perceived state of the economy at the end of the period, Perc, on the average net income, NI, of the bank.

NI	Conclusions
$(\text{Max}\downarrow \wedge \text{SC}\downarrow \wedge \text{Beta}\uparrow) \Rightarrow \frac{\partial \text{NI}}{\partial \text{Perc}}\uparrow$ or, $\frac{\partial^2 \text{NI}}{\partial \text{Max}\,\partial \text{Perc}} < 0$, and $\frac{\partial^2 \text{NI}}{\partial \text{SC}\,\partial \text{Perc}} < 0$, and $\frac{\partial^2 \text{NI}}{\partial \text{Beta}\,\partial \text{Perc}} > 0$	When the wealth is more equally distributed and the screening capacity is low, it becomes harder for the bank to lend its funds. Under these circumstances, a more severe level of perceived risk may decrease the average NI of the bank.

Table 15. The effects of the perceived state of the economy at the end of the period, Perc, on the average total amount of loans, TL.

TL	Conclusions
$\text{Beta}\uparrow \Rightarrow \frac{\partial \text{TL}}{\partial \text{Perc}}\uparrow$ or, $\frac{\partial^2 \text{TL}}{\partial \text{Beta}\,\partial \text{Perc}} > 0$	When the level of perceived risk increases, the average amount of TL decreases. This effect becomes stronger when the level of wealth equality is higher. Under these circumstances, the regulator should take into account that imposing a more stringent level of perceived risk on the bank may further decrease TL.

Table 16. The effects of the perceived state of the economy at the end of the period, Perc, on the probability that the owners' equity in the bank will be eliminated, P_r.

P_r	Conclusions
$(\text{SC}\uparrow \wedge \text{Beta}\downarrow) \Rightarrow \frac{\partial P_r}{\partial \text{Perc}}\uparrow$ or, $\frac{\partial^2 P_r}{\partial \text{SC}\,\partial \text{Perc}} > 0$, and $\frac{\partial^2 P_r}{\partial \text{Beta}\,\partial \text{Perc}} < 0$	Increasing the level of perceived risk decreases the probability that the owners' equity will be eliminated. This effect becomes stronger for a higher level of screening capacity and a lower level of Beta (higher inequality of the wealth distribution).

In consideration of the effects of Perc on P_r, our results show that increasing the level of perceived risk decreases P_r. This effect becomes stronger for a higher level of SC and a lower level of Beta (higher inequality of the wealth distribution). Table 16 summarizes the effects of Perc on P_r.

In summary, increasing the level of perceived risk decreases P_r. This effect becomes stronger when there is a higher SC and higher inequality of borrowers' wealth distribution. Regulators should tailor the proper level of stress testing to each bank given its individual SC and the distribution of wealth across its potential borrowers. When the level of wealth equality is higher, the regulator should be cautious about imposing a more stringent level of stress testing on the bank since this may lead to a further decrease in TL. In addition, if the SC of the bank is low, it becomes harder for the bank to lend its funds. Therefore, a more stringent level of perceived risk may lead to a decrease in the average NI of the bank.

Future research could evaluate the need for regulation of the loan-to-value ratio. Under what conditions would there be no difference between the level of loan-to-value ratio preferred by the regulator and that preferred by the bank owners? The need for regulation should be assessed with the regulator considering not only the probability that owners' equity could be eliminated but also other variables that might have effects on the macro economy, such as the total amount of loans, the total number of borrowers, and the net income of the bank. Another question worthy of consideration is the interaction between the bank's corporate governance and the need for external regulation on loan-to-value ratio. Finally, of particular interest would be real cross-country data testing of our model predictions.

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Appendix A. Power Law Distribution

We say that a continuous variable x is power law distributed in the range $[a, b]$ if the probability density function is $g(x) = \frac{\beta-1}{a^{1-\beta}-b^{1-\beta}} x^{-\beta}$, for $\beta > 1$. The cumulative distribution function is given by $G(x) = \frac{a^{1-\beta}-x^{1-\beta}}{a^{1-\beta}-b^{1-\beta}}$. For a discrete distribution, we can approximate the probability of x by $G(x) - G(x-1) = \frac{1}{1-N^{1-\beta}}((x-1)^{1-\beta} - x^{1-\beta})$.

Power law exponent and the upper bound of the wealth distribution

In the following two subsections, we present (i) the effect of changes in power law exponent β and the upper bound of the wealth distribution Max on the expected mean of borrowers' wealth, and (ii) the effect of changes in β on the inequality of the wealth distribution (Lorenz curve).

A.1. *The effect of changes in β and Max on the expected wealth*

Let x denote the random wealth of a potential borrower. Assuming that x is power law distributed on the interval $[1, N]$, we obtain the probability density function $g(x) = \frac{\beta-1}{1-N^{1-\beta}} x^{-\beta}$ and the cumulative distribution function $G(x) = \frac{1-x^{1-\beta}}{1-N^{1-\beta}}$, for $\beta > 1$. The expected value of x is $E(x) = \int_1^N xg(x)\,dx$. After substituting $g(x)$ we obtain $\int_1^N xg(x)\,dx = \int_1^N x \frac{\beta-1}{1-N^{1-\beta}} x^{-\beta}\,dx$. After solving the integral, we get

$$\int_1^N x \frac{\beta-1}{1-N^{1-\beta}} x^{-\beta}\,dx = \frac{\beta-1}{1-N^{1-\beta}} \frac{N^{2-\beta}-1}{2-\beta}. \tag{A.1}$$

Rearranging Eq. (A.1) yields

$$E(x) = \left(\frac{\beta-1}{\beta-2}\right)\left(\frac{1-N^{2-\beta}}{1-N^{1-\beta}}\right). \tag{A.2}$$

In the relevant ranges of our simulations ($1.5 \leq \beta \leq 2.4$ and $50{,}000 \leq N \leq 500{,}000$), the effects of changes in β and N on the mean wealth are given by the following partial derivatives:

$$\frac{\partial E(x)}{\partial \beta} < 0; \quad \frac{\partial E(x)}{\partial N} > 0 \quad \text{and} \quad \frac{\partial E(x)}{\partial N \partial \beta} < 0. \tag{A.3}$$

Table A.1. Value of the expected wealth.

N	β									
	1.5	1.6	1.7	1.8	1.9	1.999	2.1	2.2	2.3	2.4
50,000	224	112	58	31	18	11	7.27	5	4	3
100,000	316	149	71	36	19	12	7.52	5	4	3
150,000	387	175	81	39	21	12	7.66	5	4	3
200,000	447	197	89	42	22	12	7.75	5	4	3
250,000	500	215	95	44	22	12	7.83	6	4	3
300,000	548	231	100	46	23	13	7.88	6	4	3
350,000	592	246	105	47	23	13	7.93	6	4	3
400,000	632	260	110	49	24	13	7.97	6	4	3
450,000	671	272	114	50	24	13	8.01	6	4	3
500,000	707	284	117	51	24	13	8.04	6	4	3

Table A.1 presents the value of the expected wealth for the given $1.5 \leq \beta \leq 2.4$ and $50{,}000 \leq N \leq 500{,}000$.

Table A.1 shows that for $\beta > 2$, the effect of increasing the upper bound of borrowers' wealth N on the expected wealth is negligible. That is, for a low level of β, increasing N increases the expected wealth. However, when β increases, the effect of an increase in N on the expected wealth diminishes.

A.2. *The effect of on the inequality of the wealth distribution (Lorenz curve)*

The Lorenz curve depicts the percentage of the total wealth held by the lower $\theta\%$ of the population and is given by

$$L(\theta) = \frac{\int_1^{x(\theta)} xg(x)\,dx}{E(x)}, \tag{A.4}$$

where $x(\theta)$, the inverse function of $G(x)$, is given by

$$x(\theta) = [\theta(N^{1-\beta} - 1) + 1]^{\frac{1}{1-\beta}}, \tag{A.5}$$

and $E(x)$ is given by Eq. (A.2). Hence, we get

$$L(\theta) = \frac{\left(\frac{\beta-1}{\beta-2}\right)\left(\frac{1-x^{2-\beta}(\theta)}{1-N^{1-\beta}}\right)}{\left(\frac{\beta-1}{\beta-2}\right)\left(\frac{1-N^{2-\beta}}{1-N^{1-\beta}}\right)} = \frac{1 - x^{2-\beta}(\theta)}{1 - N^{2-\beta}}. \tag{A.6}$$

Thus, by substituting Eq. (A.5) in Eq. (A.6), we obtain the Lorenz curve

$$L(\theta) = \frac{1 - [\theta(1 - N^{1-\beta}) - 1]^{\frac{\beta-2}{\beta-1}}}{1 - N^{2-\beta}}. \tag{A.7}$$

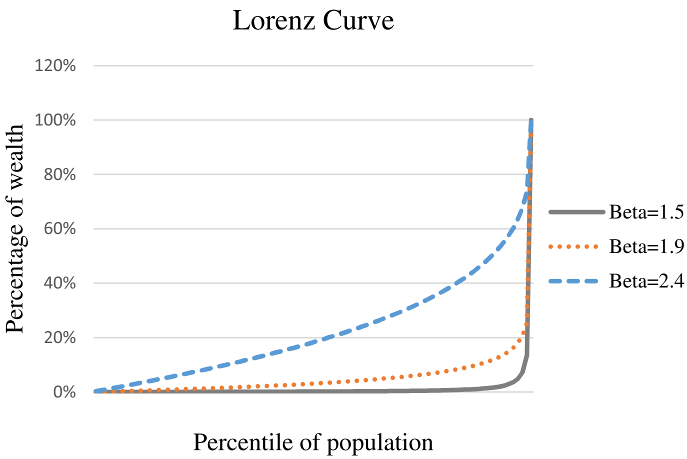

Fig. A.1. Lorenz curve for different levels of β.

As β increases, the Lorenz curve exhibits a lesser degree of wealth inequality (or a higher degree of wealth equality). For example, for a power law distribution of wealth between 1 and $N = 400{,}000$ dollars, we get the Lorenz curve depicted in Fig. A.1.

A well-known measure (Gini, 1936) of equality of wealth is the area under the Lorenz curve multiplied by 2. As the area under the curve increases, the equality of wealth increases and reaches its maximum, which is a value of 1. Figure A.1 shows that when $\beta = 1.5$, the Lorenz curve exhibits a high degree of inequality of wealth, and the area under the curve is minimal. For $\beta = 2.4$, the Lorenz curve exhibits a higher degree of equality of wealth, and the area under the curve increases.

Appendix B

With a probability $\lambda_n(\delta_P)$ the borrower will default and the bank will make only the net realized value of the collateral assets, $C_n^R(\delta_P)$. With a probability $1 - \lambda_n(\delta_P)$ there is no default and the bank collects $L_n(1 + i + \rho_n)$. Since the bank wants to obtain an effective interest of i, we get

$$\lambda_n(\delta_P)C_n^R(\delta_P) + (1 - \lambda_n(\delta_P))L_n(1 + i + \rho_n) = L_n(1 + i).$$

Thus,

$$(1 - \lambda_n(\delta_P))L_n(1 + i) + (1 - \lambda_n(\delta_P))\rho_n L_n = L_n(1 + i) - \lambda_n(\delta_P)C_n^R(\delta_P).$$

Therefore,

$$(1-\lambda_n(\delta_P))\rho_n L_n = L_n(1+i) - \lambda_n(\delta_P)C_n^R(\delta_P) - (1-\lambda_n(\delta_P))L_n(1+i),$$

and in turn,

$$(1-\lambda_n(\delta_P))\rho_n = \left((1+i) - \lambda_n(\delta_P)\frac{C_n^R(\delta_P)}{L_n} - (1-\lambda_n(\delta_P))(1+i)\right).$$

Isolating ρ_n gives

$$\begin{aligned}\rho_n &= \frac{\left((1+i) - \lambda_n(\delta_P)\frac{C_n^R(\delta_P)}{L_n} - (1-\lambda_n(\delta_P))(1+i)\right)}{1-\lambda_n(\delta_P)}\\ &= \frac{\lambda_n(\delta_P)\left(1-\frac{C_n^R(\delta_P)}{L_n}\right) + \lambda_n(\delta_P)i}{1-\lambda_n(\delta_P)},\end{aligned}$$

and consequently,

$$\rho_n = \frac{\lambda_n(\delta_P)}{1-\lambda_n(\delta_P)}\left(1-\frac{C_n^R(\delta_P)}{L_n}+i\right).$$

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